

# Probing high energy QCD via 2-particle correlations

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*Forward Physics at RHIC, BNL (2012)*

# Di-hadron correlations

## Rapidity correlations

ridge (near side)

nucleus-nucleus collisions

proton-proton collisions

## Angular correlations (away side)

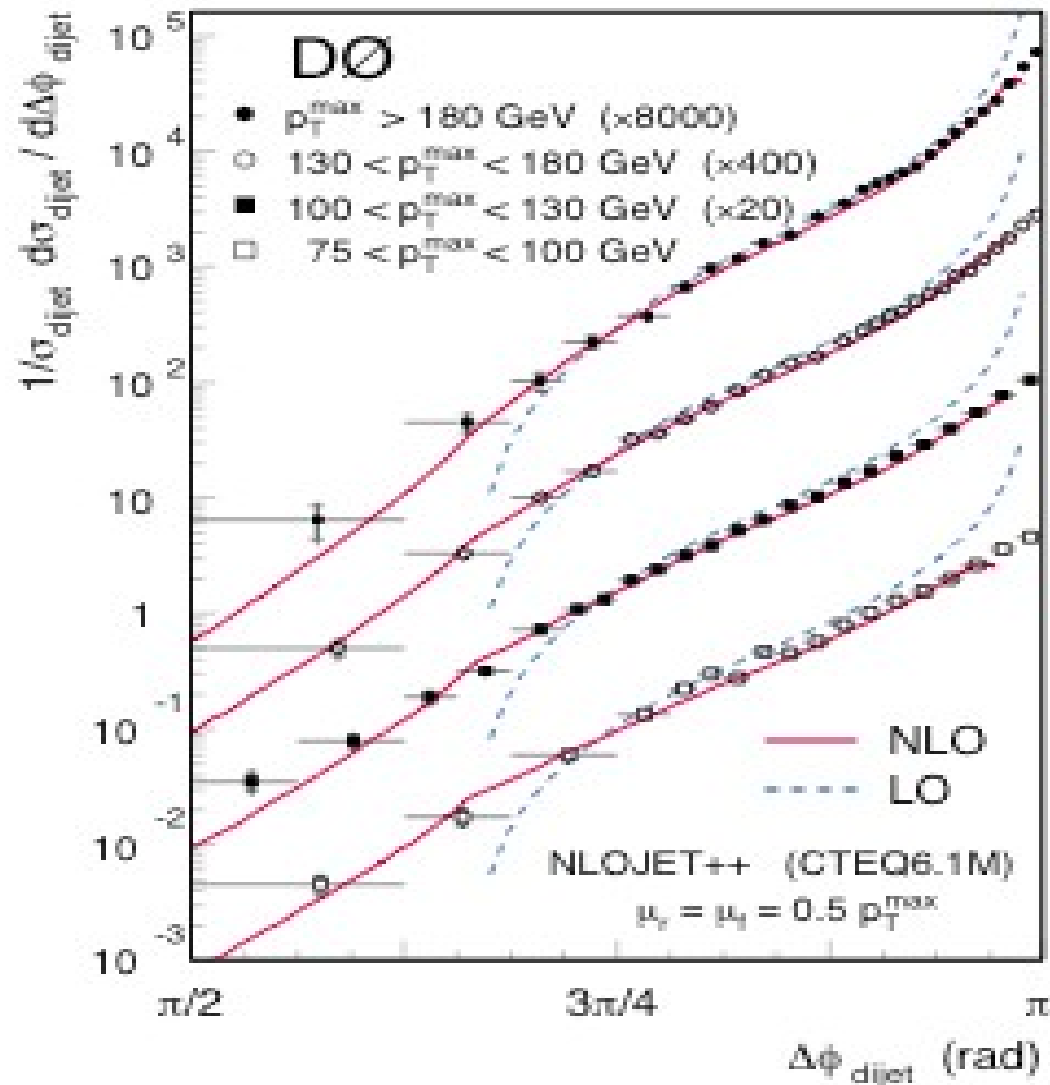
large  $x$  (high  $p_t$ ): pQCD

small  $x$ : CGC

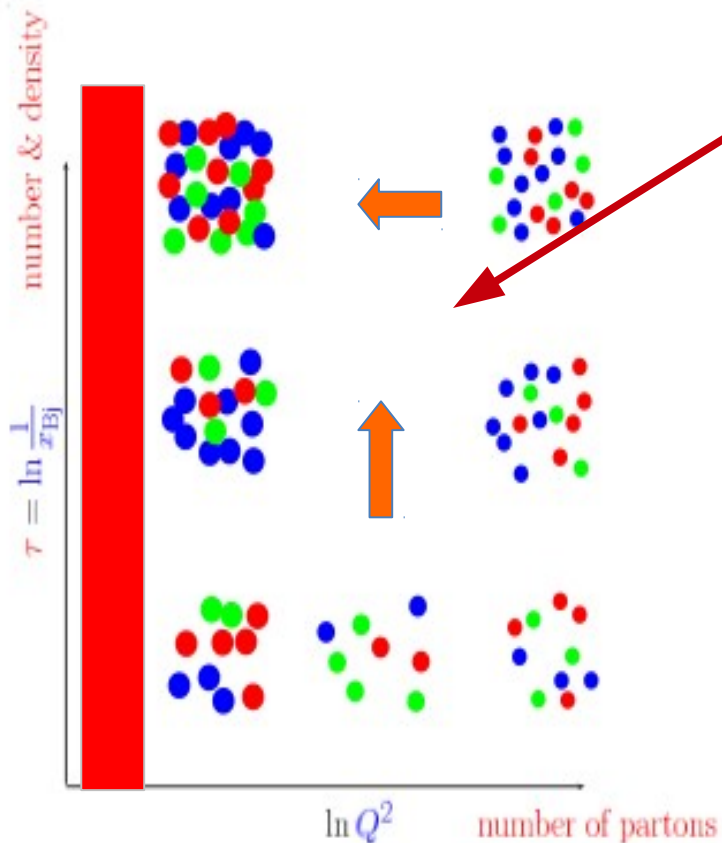
forward pA (dilute-dense) collisions

# Di-jet correlations at large x (high $p_t$ ): pQCD

di-jets are back to back



# CGC: universal gluonic matter



How does this happen ?

How do correlation functions of these evolve ?

Is there a universal fixed point for the RG evolution of d.o.f

How does the coupling run ?

How does saturation transition to chiral symmetry breaking and confinement

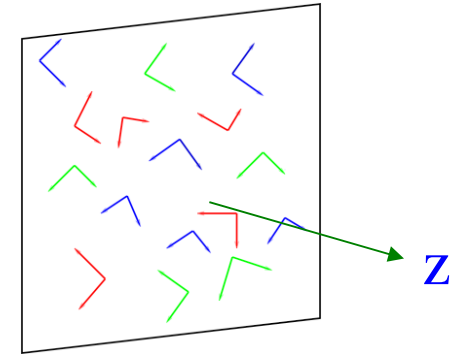
$$Q_s^2(x, b_t, A) \sim A^{1/3} \left( \frac{1}{x} \right)^{0.3}$$

# QCD at low $x$ : **CGC**

two main effects: “multiple scatterings”  
evolution with  $\ln(1/x)$

$$\mathbf{A}_a^\mu(\mathbf{x}_t, x^-) \sim \delta^{\mu+} \delta(x^-) \alpha_a(\mathbf{x}_t)$$

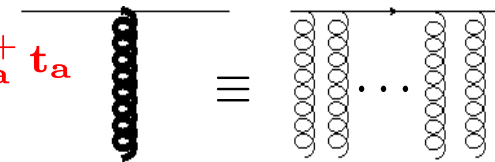
$$\alpha^a(\mathbf{k}_t) = g \rho^a(\mathbf{k}_t) / k_t^2$$



**CGC observables:**  $\langle \text{Tr } V \dots V^\dagger \rangle$  with

propagation of quarks and gluons in the  
background of the classical field

$$V(\mathbf{x}_t) = \hat{\mathbf{P}} e^{ig \int dx^- \mathbf{A}_a^+ t_a}$$



gluon distribution:  $\sim \int^{Q^2} \frac{d^2 \mathbf{k}_t}{k_t^2} \phi(\mathbf{x}, \mathbf{k}_t)$  with  $\phi(\mathbf{k}_t^2) \sim \langle \rho_a^*(\mathbf{k}_t) \rho_a(\mathbf{k}_t) \rangle$

pQCD with collinear factorization: *single scattering*  
*evolution with  $\ln Q^2$*

# JIMWLK evolution equation

*re-sum  $\ln(1/x)$*

$$\frac{d}{d \ln 1/x} \langle O \rangle = \frac{1}{2} \left\langle \int d^2 x d^2 y \frac{\delta}{\delta \alpha_x^b} \eta_{xy}^{bd} \frac{\delta}{\delta \alpha_y^d} O \right\rangle$$

$$\eta_{xy}^{bd} = \frac{1}{\pi} \int \frac{d^2 z}{(2\pi)^2} \frac{(x-z) \cdot (y-z)}{(x-z)^2 (y-z)^2} \left[ \underbrace{1 + U_x^\dagger U_y}_{\text{virtual}} - \underbrace{U_x^\dagger U_z - U_z^\dagger U_y}_{\text{real}} \right]^{bd}$$

***$U$  is a Wilson line in adjoint representation***

# **Color *Glass* Condensate**

## ***Advantages:***

*A systematic, first-principle approach to high energy scattering in QCD*

*Controlled approximations*

*Same formalism can describe a wide range of phenomena*

## ***Disadvantages:***

*Applicable at low  $x$  (high  $x$ ,  $Q^2$  missing)*

# ***Observables***

Talks by  
*J. Albacete, K. Dusling,  
Y. Kovchegov, K. Tuchin*

***DIS:***

*structure functions  
particle production*

***dilute-dense (pA, forward pp ) collisions:***

*multiplicities*

*$p_t$  spectra*

***di-hadron angular correlations***

***dense-dense (AA, pp) collisions:***

*multiplicities, spectra*

*long range rapidity correlations*

***Spin***

# 2-particle kinematics in CGC

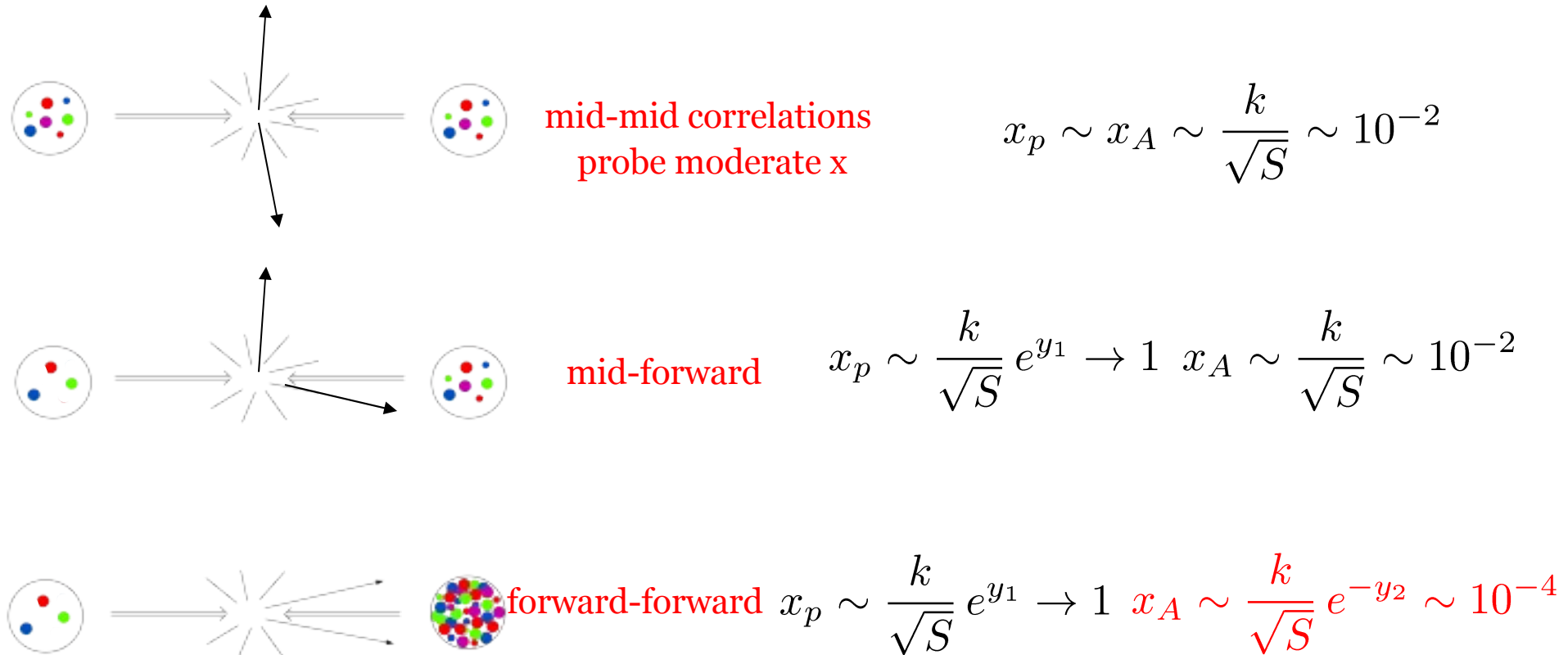
produced partons:  $k_1, y_1$   $k_2, y_2$

$$x_p = \frac{k_1 e^{y_1} + k_2 e^{y_2}}{\sqrt{s}}$$

$$x_A = \frac{k_1 e^{-y_1} + k_2 e^{-y_2}}{\sqrt{s}}$$

scanning the wave-functions (RHIC)

$$k_1 \sim k_2 \sim k \sim 2 \text{ GeV}$$



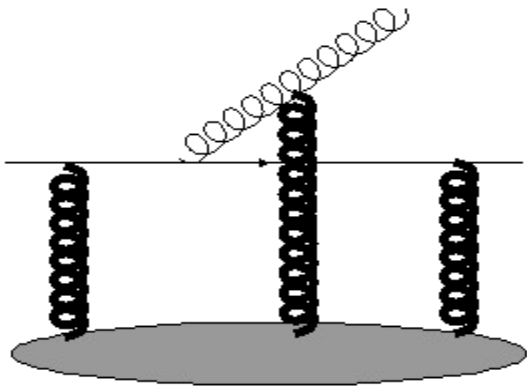
# Di-jet production: pA

J. Jalilian-Marian, Y. Kovchegov  
PRD70 (2004) 114017

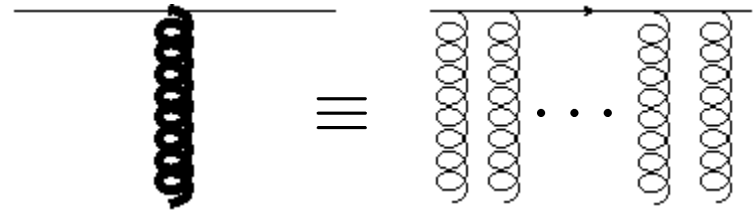
$$q(p) \text{ T} \rightarrow q(q) g(k) \text{ X}$$

LSZ reduction formalism

$$\langle q(q) g(k)_{out} | q(p)_{in} \rangle = \langle 0_{out} | a_{out}(k) b_{out}(q) b_{in}^\dagger(p) | 0_{in} \rangle$$



*with*



$$\begin{aligned} \mathcal{M}(q, k; p) &= g \int d^4x d^4y d^4z d^4r d^4\bar{r} e^{i(q \cdot z + k \cdot r - p \cdot y)} \\ &\quad \bar{u}(q) [i \overrightarrow{\not{\partial}}_z] S_F(z, x) \gamma^\nu t^c S_F(x, y) [i \overleftarrow{\not{\partial}}_y] u(p) \\ &\quad G_{\nu\rho}^{cb}(x, \bar{r}) D_{ba}^{\rho\mu}(\bar{r}, r) \epsilon_\mu(k) \end{aligned}$$

# Di-jet production: pA $q(p) T \rightarrow q(q) g(k) X$

$$\mathcal{M}(q, k; p) = g \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{d^4 k_3}{(2\pi)^4} \\ \bar{u}(q) \overrightarrow{\not{q}} S_F(q, k_1) \gamma^\nu t^c S_F(k_2, p) \overleftarrow{\not{p}} u(p) \\ G_{\nu\rho}^{cb}(k_2 - k_1, k_3) D_{ba}^{\rho\mu}(k_3, k) \epsilon_\mu(k)$$

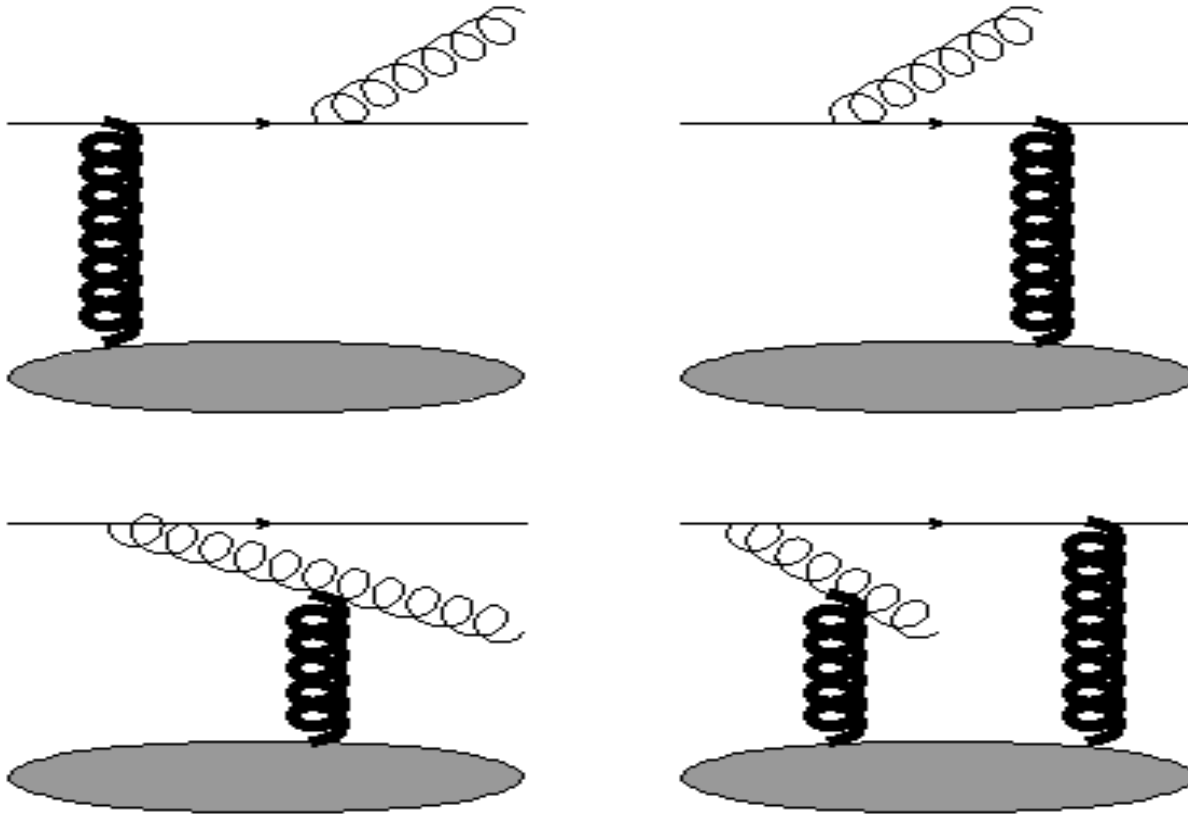
## propagators

$$S_F(q, p) \equiv (2\pi)^4 \delta^4(p - q) S_F^0(p) + S_F^0(q) \tau_f(q, p) S_F^0(p) \\ G^{\mu\nu}(q, p) \equiv (2\pi)^4 \delta^4(p - q) G^{0\mu\nu}(p) + G_\rho^{0\mu}(q) \tau_g(q, p) G^{0\rho\nu}(p)$$

## with

$$\tau_f(q, p) \equiv (2\pi) \delta(p^- - q^-) \gamma^- \int d^2 x_t e^{i(q_t - p_t) \cdot x_t} [V(x_t) - 1] \\ \tau_g(q, p) \equiv 2p^- (2\pi) \delta(p^- - q^-) \int d^2 x_t e^{i(q_t - p_t) \cdot x_t} [U(x_t) - 1]$$

# Di-jet production: pA $q(p) T \rightarrow q(q) g(k)$



$$d\sigma \sim \int \mathbf{K} \otimes \left[ \langle \text{Tr} \mathbf{V} \mathbf{V}^\dagger \rangle + \langle \text{Tr} \mathbf{V} \mathbf{V}^\dagger \mathbf{V} \mathbf{V}^\dagger \rangle + \dots \right]$$

$$\mathbf{V} \equiv \text{[Diagram of a quark line interacting with a nucleus]} \equiv \text{[Diagram of multiple gluon lines]} \dots \sim \mathbf{1} + \mathbf{O}(g \rho) + \mathbf{O}(g^2 \rho^2)$$

# Di-jet production: pA

*recall DIS, single inclusive production in pA probe dipoles*

$$< \text{Tr } V V^\dagger >$$

*di-jet production in pA (and DIS) probe quadrupoles*

*This would be problematic in pQCD*

$$< \text{Tr } V V^\dagger V V^\dagger >$$

*momentum space:*

*J. Jalilian-Marian, Y. Kovchegov, PRD70 (2004) 114017*

*coordinate space:*

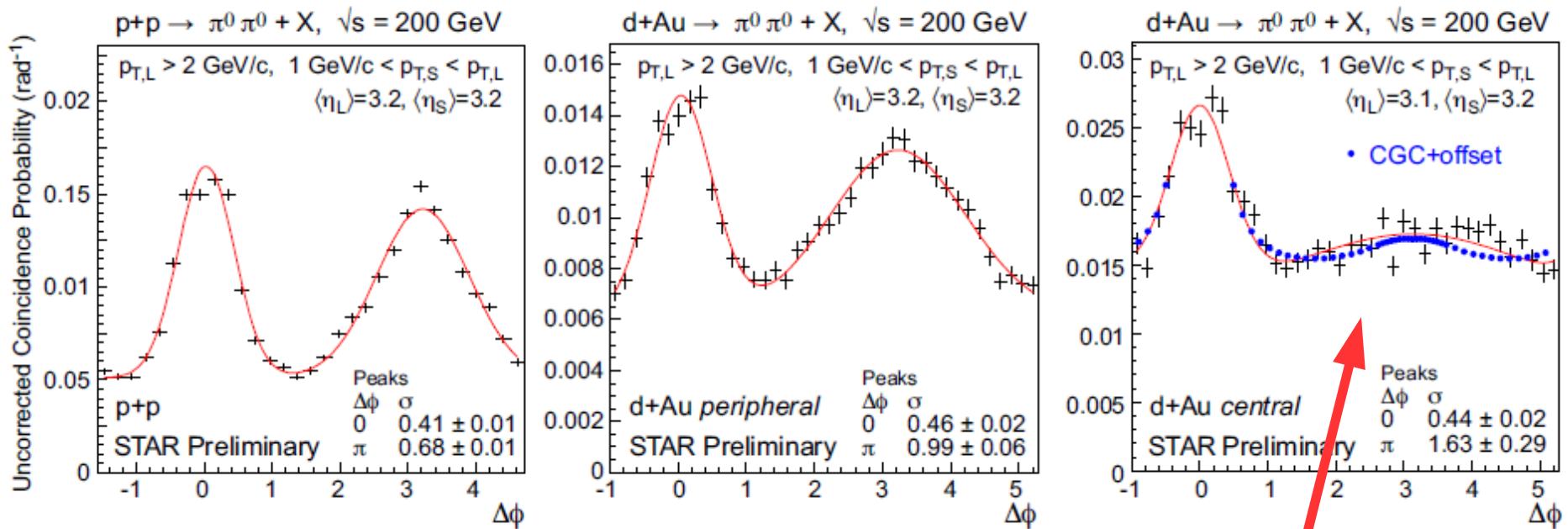
*C. Marquet, NPA796 (2007) 41*

*including gluons in the projectile*

*F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan,  
PRD83 (2011) 105005*

# disappearance of back to back jets

Recent STAR measurement (arXiv:1008.3989v1):

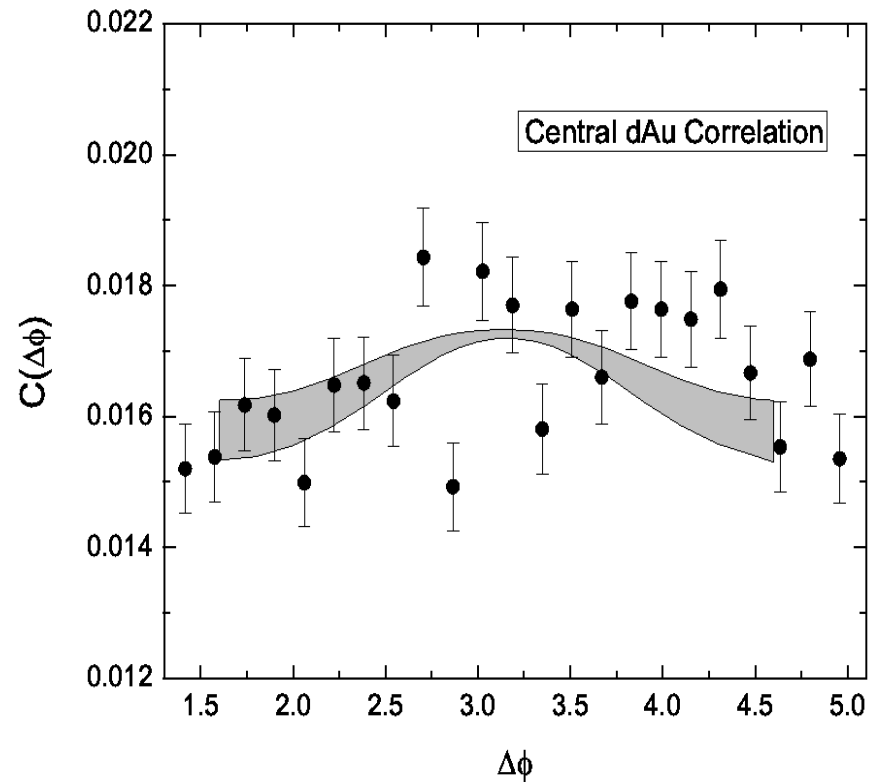
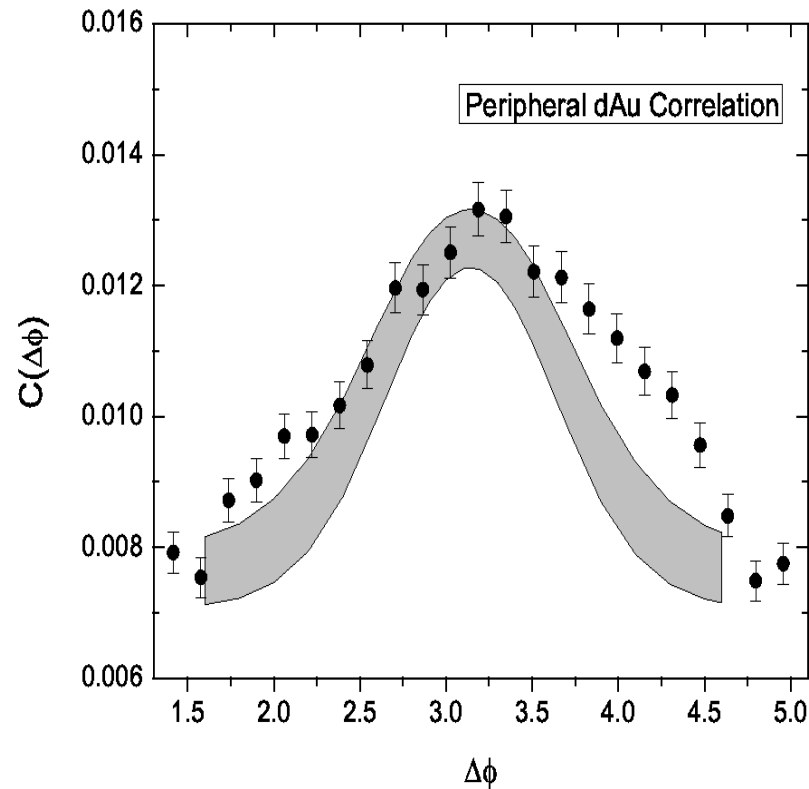


*CGC fit from*

*Albacete + Marquet, PRL (2010)  
using running coupling BK solution,  
Also by Tuchin, NPA846 (2010)*

*multiple scatterings  
de-correlate the hadrons*

# disappearance of back to back jets



*CGC fit from*  
*A. Stasto, B-W. Xiao, F. Yuan, arXiv:1109.1817*

*alternative idea: shadowing + energy loss (M. Strikman et al.)*  
*Z. Kang, I. Vitev and H. Xing, PRD85 (2012) 054024*

# Probing for Saturation Effects in Hadron-Hadron Correlations in d+Au with the Forward MPC

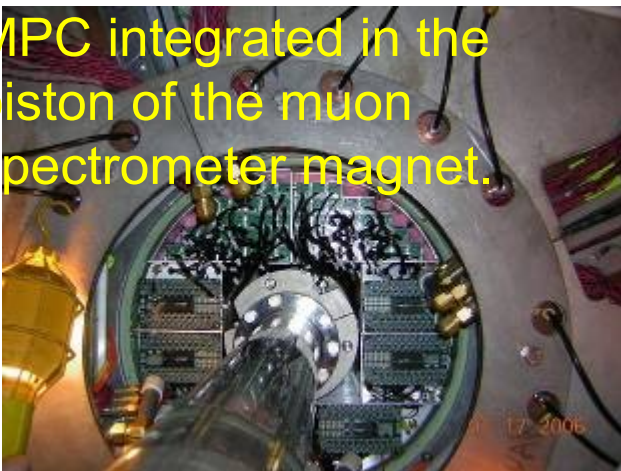
Beau Meredith, Phys.Rev.Lett. 107 (2011) 172301

## Experimental signature:

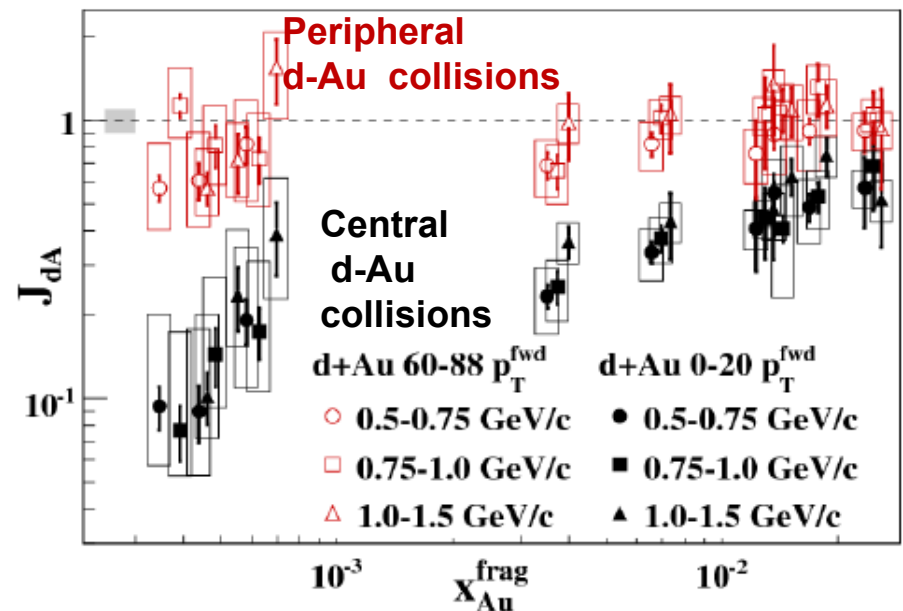
- Broadening in the correlation of back-to-back jets or hadrons
- Suppression of away-side jets or hadrons

The MPC is an NSF funded UIUC built forward EMC based on  $\text{PbWO}_4$  Crystals with APD readout.

MPC integrated in the piston of the muon spectrometer magnet.



$$J_{dA} \sim \frac{N_{dAu}^{\text{back-to-back}} / N_{\text{N-N-collision}}}{N_{pp}^{\text{back-to-back}}}$$



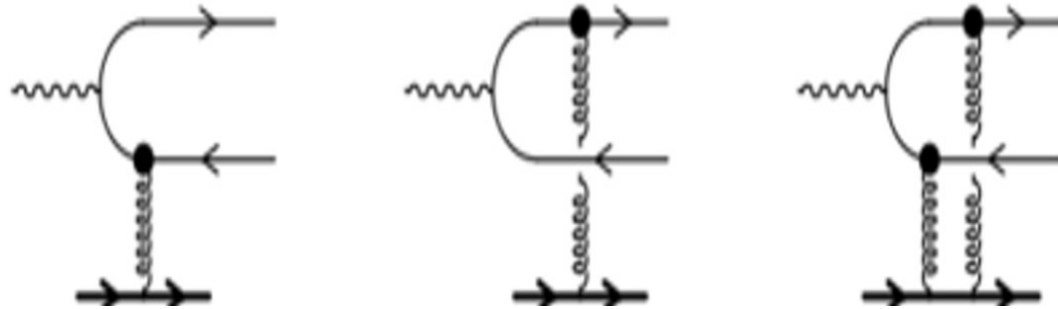
Away Side hadrons/jets in central d-Au  
suppressed by factor 5 at  $x \sim 0.0005$   
Mono-jets !? CGC ?

# Di-jet correlations in DIS

*EIC*

$$\gamma^* p(A) \rightarrow q \bar{q} X$$

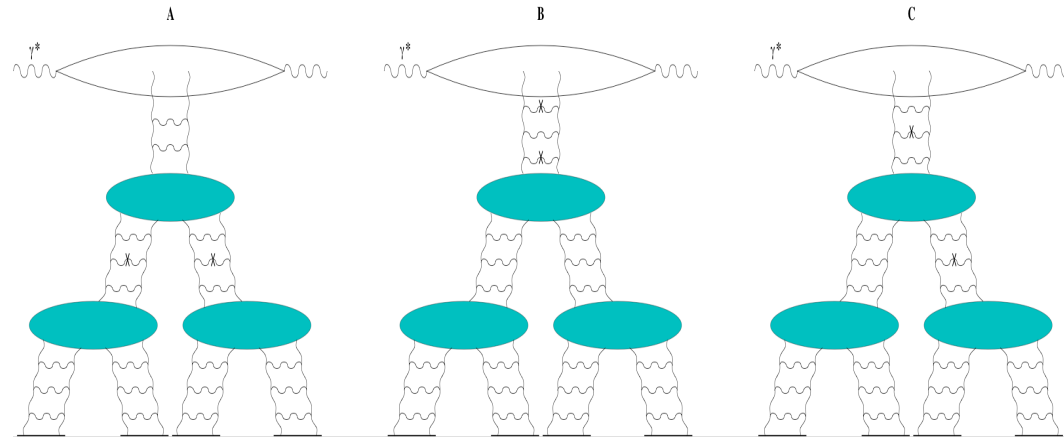
FG & JJM, PRD67 (2003)



$$\gamma^* p(A) \rightarrow g g X$$

JJM & YK, PRD70 (2004)

AK & ML, JHEP (2006)



*di-jet production in pA and DIS  
probes quadrupoles*

# di-jet production in pA

$O_2(r, \bar{r}) \equiv \text{Tr} V_r V_{\bar{r}}^\dagger$  **dipole**  $\longrightarrow$  **F2 in DIS, single hadron in pA**

$$O_4(r, \bar{r} : s) \equiv \text{Tr} V_r^\dagger t^a V_{\bar{r}} t^b [U_s]^{ab} = \frac{1}{2} \left[ \text{Tr} V_r^\dagger V_s \text{Tr} V_{\bar{r}} V_s^\dagger - \frac{1}{N_c} \text{Tr} V_r^\dagger V_{\bar{r}} \right]$$

$$O_6(r, \bar{r} : s, \bar{s}) \equiv \text{Tr} V_r V_{\bar{r}}^\dagger t^a t^b [U_s U_{\bar{s}}^\dagger]^{ba} = \frac{1}{2} \left[ \text{Tr} V_r V_{\bar{r}}^\dagger V_{\bar{s}} V_s^\dagger \text{Tr} V_s V_{\bar{s}}^\dagger - \frac{1}{N_c} \text{Tr} V_r V_{\bar{r}}^\dagger \right]$$

**quadrupole**

calculations: classical

how about quantum corrections (energy dependence) ?

energy (rapidity) dependence from JIMWLK evolution of O's  
evolution of a dipole is well known: BK eq.

***how does a quadrupole evolve?***

Mean field + large  $N_c$  : Balitsky-Kovchegov eq.

$$\frac{d}{dy} S(\mathbf{r} - \bar{\mathbf{r}}) = \frac{\bar{\alpha}_s}{2\pi} \int d^2z \frac{(\mathbf{r} - \bar{\mathbf{r}})^2}{(\mathbf{r} - \mathbf{z})^2 (\bar{\mathbf{r}} - \mathbf{z})^2} [S(\mathbf{r} - \mathbf{z}) S(\bar{\mathbf{r}} - \mathbf{z}) - S(\mathbf{r} - \bar{\mathbf{r}})]$$

$$\text{with } S(\mathbf{r} - \bar{\mathbf{r}}) \equiv \frac{1}{N_c} < \text{Tr } \mathbf{V}(\mathbf{r}) \mathbf{V}^\dagger(\bar{\mathbf{r}}) > \quad \text{and}$$

$$dP_{d \rightarrow d d} = \frac{\bar{\alpha}_s}{2\pi} \frac{(\mathbf{r} - \bar{\mathbf{r}})^2}{(\mathbf{r} - \mathbf{z})^2 (\bar{\mathbf{r}} - \mathbf{z})^2} d^2z \quad \text{dipole splitting probability}$$

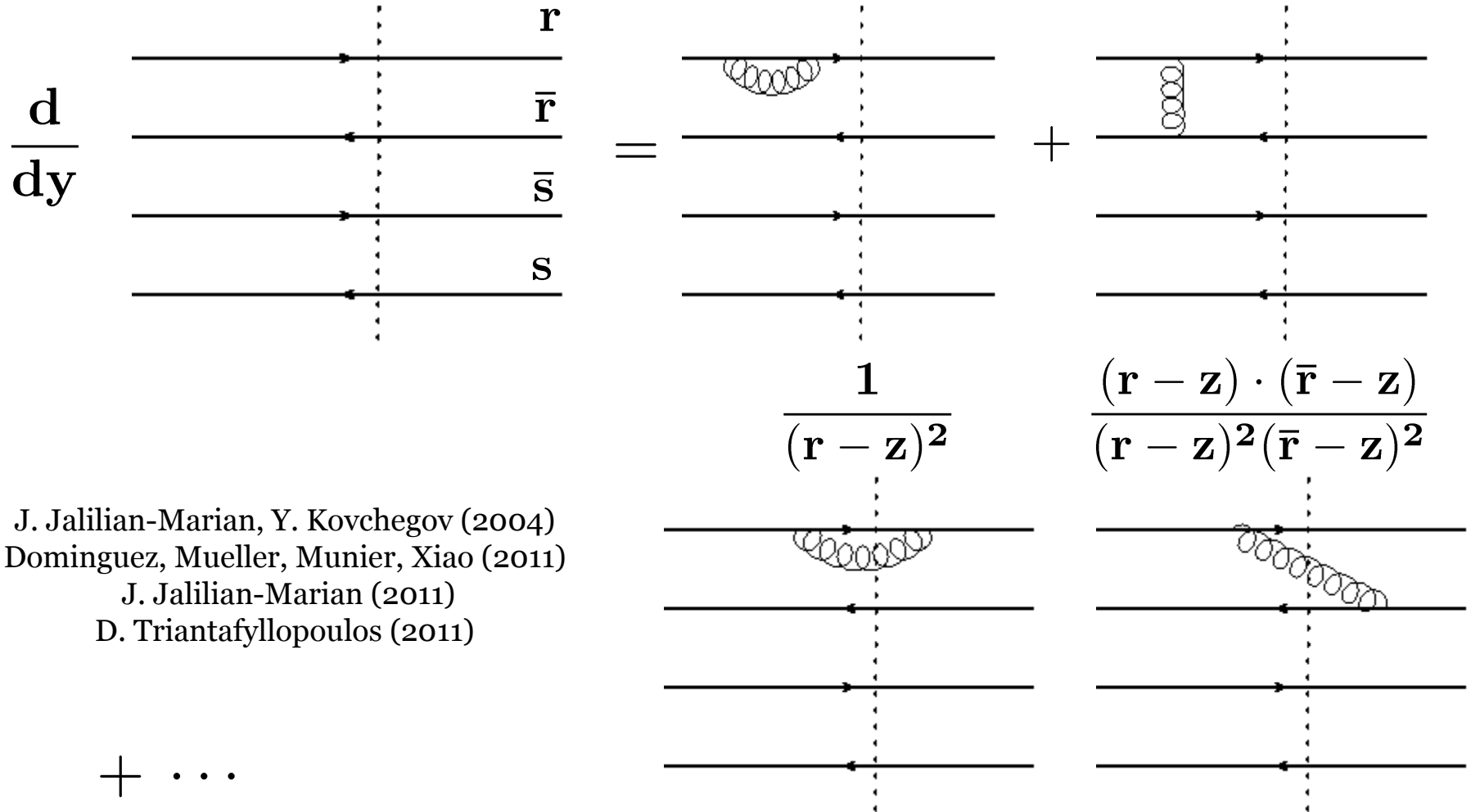
*all  $n$ -point correlators are expressed in terms of the dipoles*

**NLO:** Balitsky-Kovchegov-Weigert-Gardi-Chirilli (2007-2008)

# Evolution of quadrupole from JIMWLK

$$Q(\mathbf{r}, \bar{\mathbf{r}}, \bar{\mathbf{s}}, \mathbf{s}) \equiv \frac{1}{N_c} \langle \text{Tr } V(\mathbf{r}) V^\dagger(\bar{\mathbf{r}}) V(\bar{\mathbf{s}}) V^\dagger(\mathbf{s}) \rangle$$

radiation kernels  
as in dipole



J. Jalilian-Marian, Y. Kovchegov (2004)  
Dominguez, Mueller, Munier, Xiao (2011)

J. Jalilian-Marian (2011)  
D. Triantafyllopoulos (2011)

# Evolution of quadrupole from JIMWLK

$$\begin{aligned}
 & \frac{d}{dy} \langle Q(r, \bar{r}, \bar{s}, s) \rangle \\
 = & \frac{N_c \alpha_s}{(2\pi)^2} \int d^2 z \left\{ \left\langle \left[ \frac{(r - \bar{r})^2}{(r - z)^2 (\bar{r} - z)^2} + \frac{(r - s)^2}{(r - z)^2 (s - z)^2} - \frac{(\bar{r} - s)^2}{(\bar{r} - z)^2 (s - z)^2} \right] Q(z, \bar{r}, \bar{s}, s) S(r, z) \right. \right. \\
 + & \left[ \frac{(r - \bar{r})^2}{(r - z)^2 (\bar{r} - z)^2} + \frac{(\bar{r} - \bar{s})^2}{(\bar{r} - z)^2 (\bar{s} - z)^2} - \frac{(r - \bar{s})^2}{(r - z)^2 (\bar{s} - z)^2} \right] Q(r, z, \bar{s}, s) S(z, \bar{r}) \\
 + & \left[ \frac{(\bar{r} - \bar{s})^2}{(\bar{r} - z)^2 (\bar{s} - z)^2} + \frac{(s - \bar{s})^2}{(s - z)^2 (\bar{s} - z)^2} - \frac{(\bar{r} - s)^2}{(s - z)^2 (\bar{r} - z)^2} \right] Q(r, \bar{r}, z, s) S(\bar{s}, z) \\
 + & \left[ \frac{(r - s)^2}{(r - z)^2 (s - z)^2} + \frac{(s - \bar{s})^2}{(s - z)^2 (\bar{s} - z)^2} - \frac{(r - \bar{s})^2}{(r - z)^2 (\bar{s} - z)^2} \right] Q(r, \bar{r}, \bar{s}, z) S(z, s) \\
 - & \left[ \frac{(r - \bar{r})^2}{(r - z)^2 (\bar{r} - z)^2} + \frac{(s - \bar{s})^2}{(s - z)^2 (\bar{s} - z)^2} + \frac{(r - s)^2}{(r - z)^2 (s - z)^2} + \frac{(\bar{r} - \bar{s})^2}{(\bar{r} - z)^2 (\bar{s} - z)^2} \right] Q(r, \bar{r}, \bar{s}, s) \\
 - & \left[ \frac{(r - s)^2}{(r - z)^2 (s - z)^2} + \frac{(\bar{r} - \bar{s})^2}{(\bar{r} - z)^2 (\bar{s} - z)^2} - \frac{(\bar{r} - s)^2}{(\bar{r} - z)^2 (s - z)^2} - \frac{(r - \bar{s})^2}{(r - z)^2 (\bar{s} - z)^2} \right] S(r, s) S(\bar{r}, \bar{s}) \\
 - & \left. \left[ \frac{(r - \bar{r})^2}{(r - z)^2 (\bar{r} - z)^2} + \frac{(s - \bar{s})^2}{(s - z)^2 (\bar{s} - z)^2} - \frac{(r - \bar{s})^2}{(r - z)^2 (\bar{s} - z)^2} - \frac{(\bar{r} - s)^2}{(\bar{r} - z)^2 (s - z)^2} \right] S(r, \bar{r}) S(\bar{s}, s) \right\rangle \Bigg\}
 \end{aligned}$$

$$\frac{d}{dy} Q = \int P_1 [Q S] - P_2 [Q] + P_3 [S S] \quad \text{with} \quad P_1 - P_2 + P_3 = 0$$

“approximate solution”: Iancu-Triantafyllopoulos, arXiv:1109.0302

# quadrupole evolution in the linear regime

define  $\mathbf{T}(\mathbf{r}, \bar{\mathbf{r}}) \equiv 1 - \mathbf{S}(\mathbf{r}, \bar{\mathbf{r}})$        $\mathbf{T}_Q(\mathbf{r}, \bar{\mathbf{r}}, \bar{\mathbf{s}}, \mathbf{s}) \equiv 1 - \mathbf{Q}(\mathbf{r}, \bar{\mathbf{r}}, \bar{\mathbf{s}}, \mathbf{s})$

re-write the evolution eq. for  $\mathbf{T}_Q$  rather than  $\mathbf{Q}$

expand in powers of gauge fields (or color charges)

ignore contribution of non-linear terms:  $\mathbf{T} \mathbf{T}$  and  $\mathbf{T}_Q \mathbf{T}$

$$\mathcal{O}(\alpha^2) \quad \mathbf{T}_Q(\mathbf{r}, \bar{\mathbf{r}}, \bar{\mathbf{s}}, \mathbf{s}) \rightarrow \mathbf{T}(\mathbf{r}, \bar{\mathbf{r}}) + \mathbf{T}(\mathbf{r}, \mathbf{s}) + \dots$$

with  $\mathbf{T}(\mathbf{r}, \bar{\mathbf{r}}) \sim \alpha^2(\mathbf{r}, \bar{\mathbf{r}})$

**quadrupole evolution reduces to a sum of BFKL evolution eqs**

Dominguez, Mueller, Munier, Xiao (2011)

J. Jalilian-Marian (2011)

D. Triantafyllopoulos (2011)

# di-hadron correlations in the high $p_t$ limit

$\mathcal{O}(\alpha^2)$

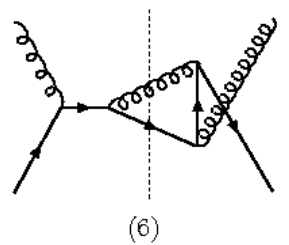
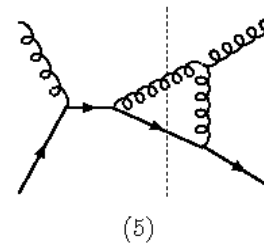
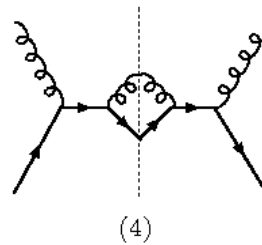
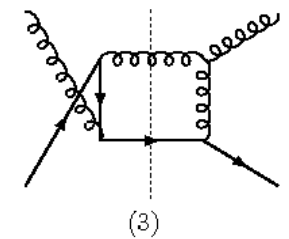
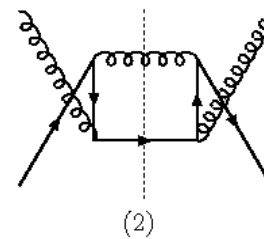
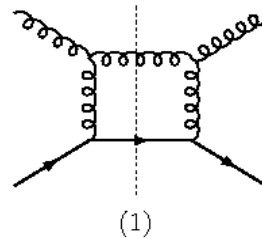
Dominguez, Marquet, Xiao, Yuan (2011)

Dominguez, Xiao, Yuan (2011)

*factorization of target distribution functions and  
hard scattering matrix element*

$$d\sigma \sim \Phi \otimes \frac{d\sigma^{2 \rightarrow 2}}{dt}$$

$$\begin{aligned} \frac{d\sigma^{qg \rightarrow qg}}{dt} &\sim \frac{1}{s^2} \left[ \frac{4}{9} \frac{s^2 + u^2}{-su} \right. \\ &+ \left. \frac{s^2 + u^2}{t^2} \right] \end{aligned}$$



**partons are back to back**

# quadrupole evolution in the linear regime

$\mathcal{O}(\alpha^4)$

momentum space

define

$$\hat{T}_4(l_1, l_2, l_3, l_4) \equiv \frac{1}{N_c} \text{Tr } \rho(l_1) \rho(l_2) \rho(l_3) \rho(l_4)$$

assume  $l_1 \neq l_2 \neq l_3 \neq l_4$  subject to an overall delta function

contribution only from linear term in expansion of Wilson lines  
(except for the z-dependent ones)

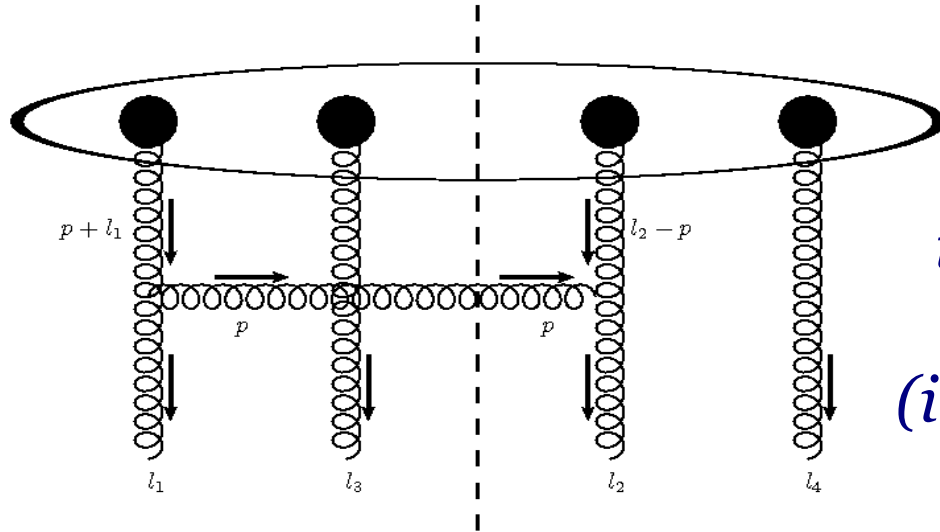
quadrupole evolution eq. reduces to Bartels-Jaroszewicz-Kwiecinski-Praszalowicz (BJKP) eq. for evolution of 4-Reggeized gluons in a singlet state

# quadrupole evolution in the linear regime

**BJKP equation**

$\mathcal{O}(\alpha^4)$  : 4-gluon exchange

*J. Jalilian-Marian, PRD85 (2012) 014037*



*the color structure is identical  
on both sides of this eq.  
(independent of color averaging)*

$$\begin{aligned} \frac{d}{dy} \hat{T}_4(l_1, l_2, l_3, l_4) &= \frac{N_c \alpha_s}{\pi^2} \int d^2 p_t \left[ \frac{p^i}{p_t^2} - \frac{(p^i - l_1^i)}{(p_t + l_1)^2} \right] \cdot \left[ \frac{p^i}{p_t^2} - \frac{(p^i - l_2^i)}{(p_t + l_2)^2} \right] \\ &\quad \hat{T}_4(p_t + l_1, l_2 - p_t, l_3, l_4) + \dots \\ &- \frac{N_c \alpha_s}{(2\pi)^2} \int d^2 p_t \left[ \frac{l_1^2}{p_t^2 (l_1 - p_t)^2} + \{l_1 \rightarrow l_2, l_3, l_4\} \right] \hat{T}_4(l_1, l_2, l_3, l_4) \end{aligned}$$

**this will de-correlate the produced partons at high  $p_t > Q_s$**

# color structure

$$\hat{\mathbf{T}}_4(\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3, \mathbf{l}_4) \equiv \frac{1}{N_c} \text{Tr} \rho(\mathbf{l}_1) \rho(\mathbf{l}_2) \rho(\mathbf{l}_3) \rho(\mathbf{l}_4) = \text{Tr} (t^a t^b t^c t^d) \rho^a(\mathbf{l}_1) \rho^b(\mathbf{l}_2) \rho^c(\mathbf{l}_3) \rho^d(\mathbf{l}_4)$$

$$\begin{aligned} \text{Tr} (t^a t^b t^c t^d) &= \frac{1}{4N_c} [\delta^{ab} \delta^{cd} - \delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc}] \\ &+ \frac{1}{8} [d^{abr} d^{cdr} - d^{acr} d^{bdr} + d^{adr} d^{bcr}] \\ &+ \frac{i}{8} [d^{abr} f^{cdr} - d^{acr} f^{bdr} + d^{adr} f^{bcr}] \end{aligned}$$

**overall state is a singlet, how about pairwise?**

**for  $N_c = 3$**

$$[\delta^{ab} \delta^{cd} + \delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc}] = 3 [d^{abr} d^{cdr} + d^{acr} d^{bdr} + d^{adr} d^{bcr}]$$

**can the exchanged pairs be in a bound state?**

**J. Bartels: YES!**

# the linear regime

$\mathcal{O}(\alpha^3)$  : 3-gluon (odderon) exchange

$V V^\dagger V$

*Hatta, Iancu, Itakura, McLerran  
Kovchegov et al.*

**BJKP equation**

**BJKP equation describes evolution of n-Reggeized  
gluons in a singlet state**

**JIMWLK (linear) and BJKP eqs. agree for n=2,3,4**

**non-linear interactions:**

- 1) MV action with JIMWLK evolution**
- 2) Triple (and more) pomeron vertices**

*Chirilli, Szymanowski, Wallon (2010)*

# QCD at high energy

## Two distinct approaches:

1) CGC

*McLerran-Venugopalan effective action*  
*JIMWLK evolution*

2) Reggeized-gluon exchange

*BJKP equation*  
*triple pomeron vertex*

***Conjecture: CGC contains BJKP + multi-pomeron vertices***

## quadrupole evolution: limits

$$\langle Q(r, \bar{r}, \bar{s}, s) \rangle \equiv \frac{1}{N_c} \langle \text{Tr} V(r) V^\dagger(\bar{r}) V(\bar{s}) V^\dagger(s) \rangle$$

**can be calculated in a Gaussian model DMXY**

**line config.:**

$$r = \bar{s}, \bar{r} = s, z \equiv r - \bar{r}$$

**square config.:**

$$r - \bar{s} = \bar{r} - s = r - \bar{r} = \dots \equiv z$$

**“naive” Gaussian:**

$$Q = S^2$$

**Gaussian**

$$Q_{\parallel}(z) \approx \frac{N_c + 1}{2} [S(z)]^{2 \frac{N_c + 2}{N_c + 1}} - \frac{N_c - 1}{2} [S(z)]^{2 \frac{N_c - 2}{N_c - 1}}$$

**Gaussian + large  $N_c$**

$$Q_{\parallel}(z) \rightarrow S^2(z) [1 + 2 \log[S(z)]]$$

# quadrupole evolution: limits

$$\langle Q(r, \bar{r}, \bar{s}, s) \rangle \equiv \frac{1}{N_c} \langle \text{Tr} V(r) V^\dagger(\bar{r}) V(\bar{s}) V^\dagger(s) \rangle$$

## Gaussian

$$Q_{sq}(z) = [S(z)]^2 \left[ \frac{N_c + 1}{2} \left( \frac{S(z)}{S(\sqrt{2}z)} \right)^{\frac{2}{N_c+1}} - \frac{N_c - 1}{2} \left( \frac{S(\sqrt{2}z)}{S(z)} \right)^{\frac{2}{N_c-1}} \right]$$

## Gaussian + large $N_c$

$$Q_{sq}(z) = \left[ 1 + 2 \ln \left( \frac{S(z)}{S(\sqrt{2}z)} \right) \right]$$

# quadrupole evolution on lattice

*a “random” (Gaussian) distribution of color charges (at initial rapidity  $y_0$ )*

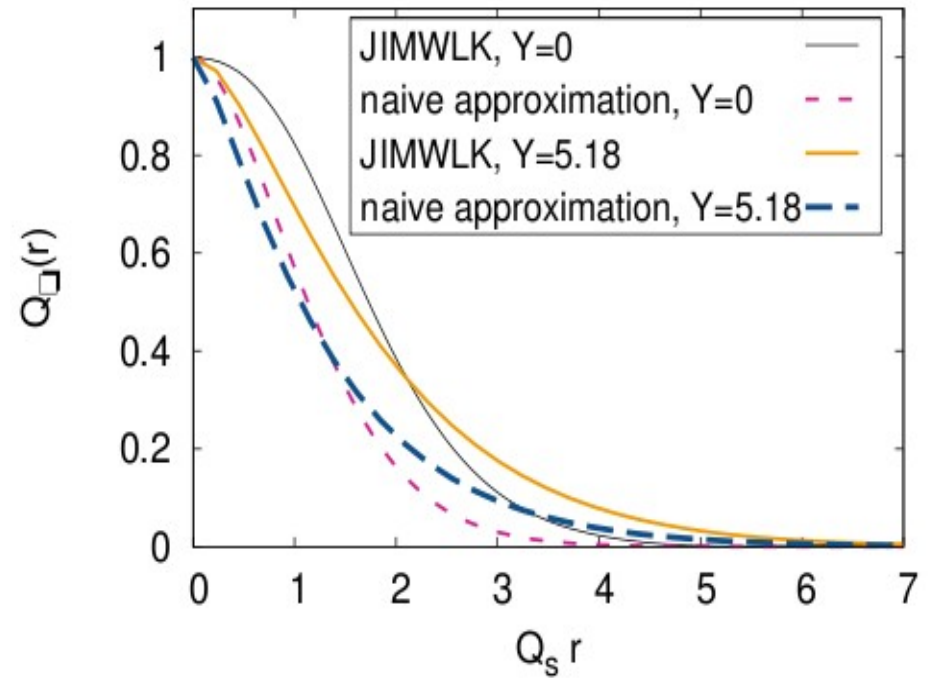
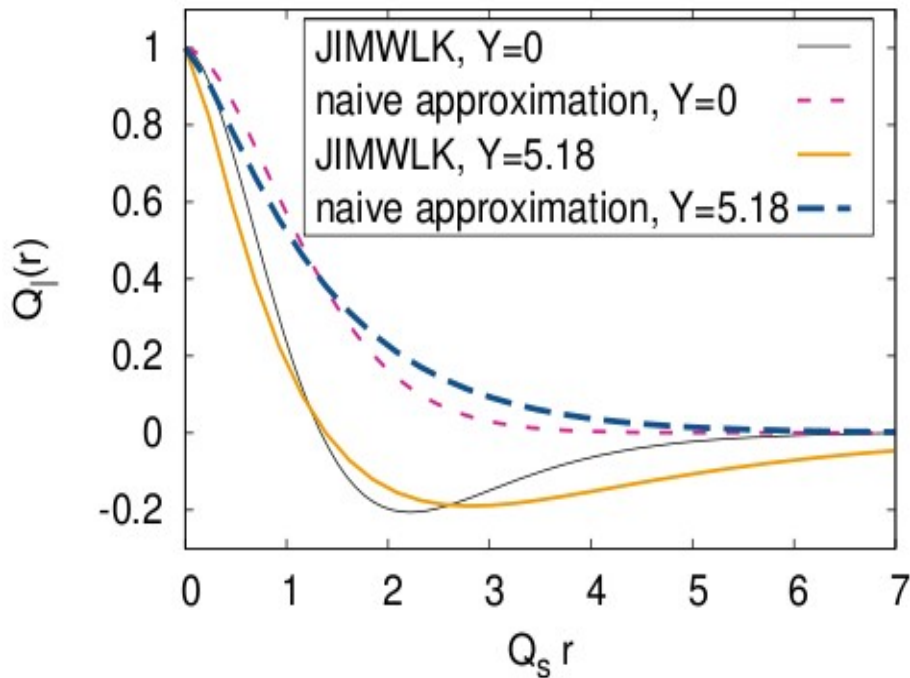
*construct the Wilson line*

*evolve the Wilson line to a higher rapidity  $y$*

*compute ensemble average of any number of Wilson lines at  $y$*

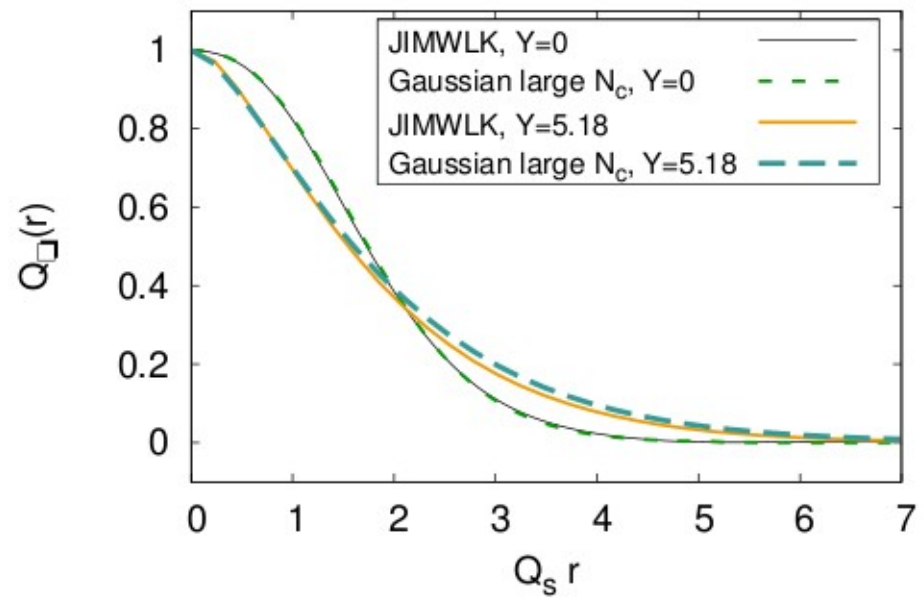
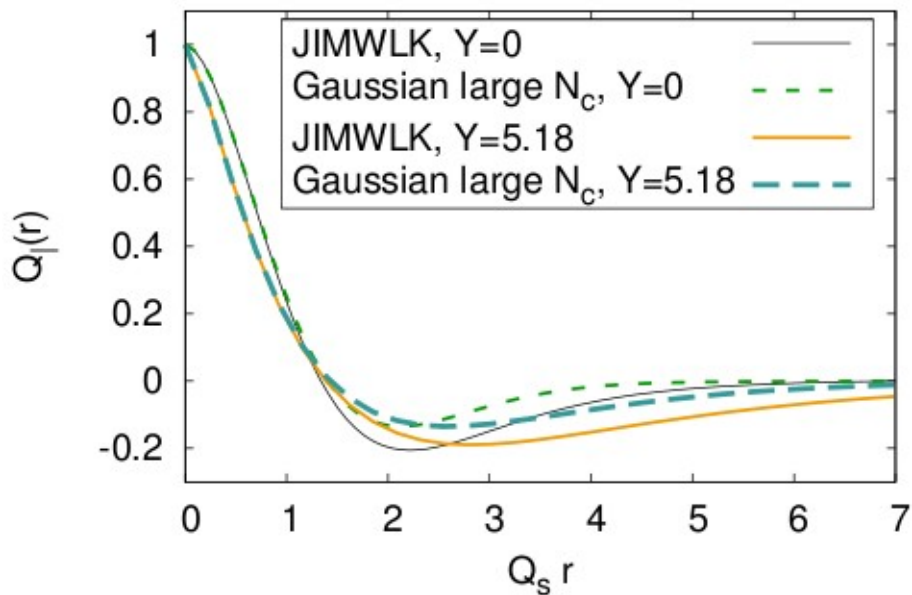
# Quadrupole evolution

comparing with “naive” Gaussian



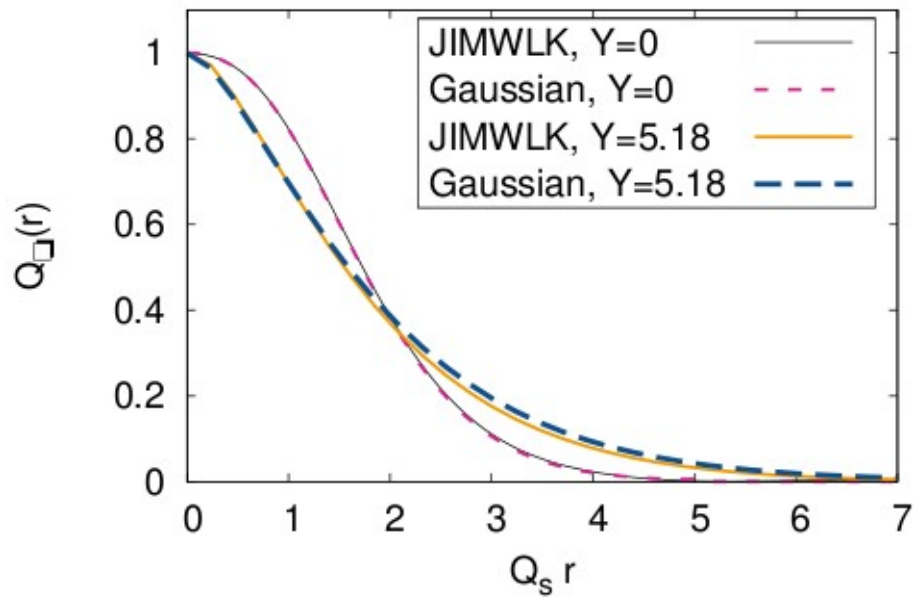
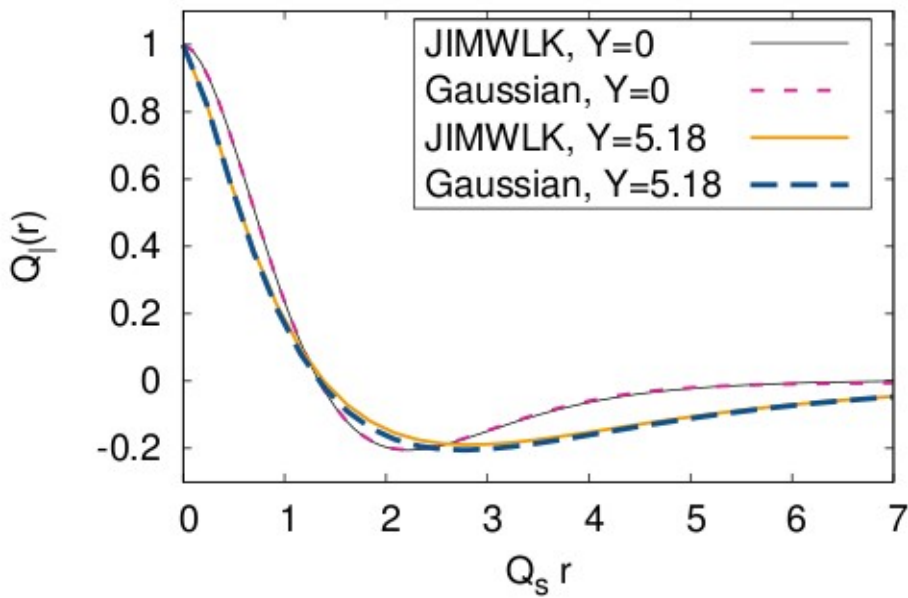
# Quadrupole evolution

comparing with Gaussian + large  $N_c$

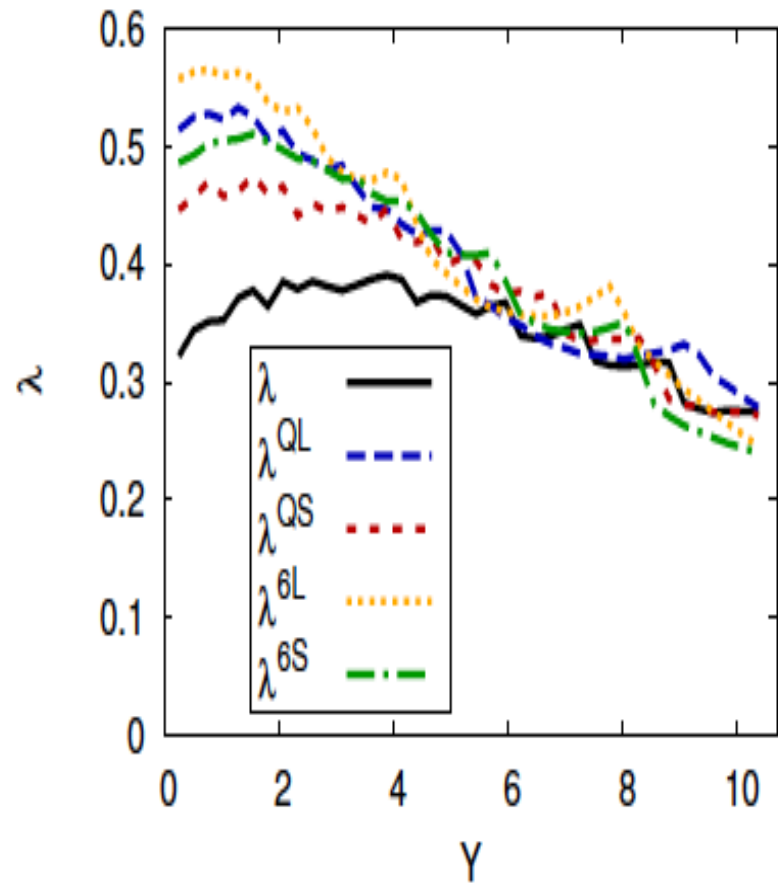
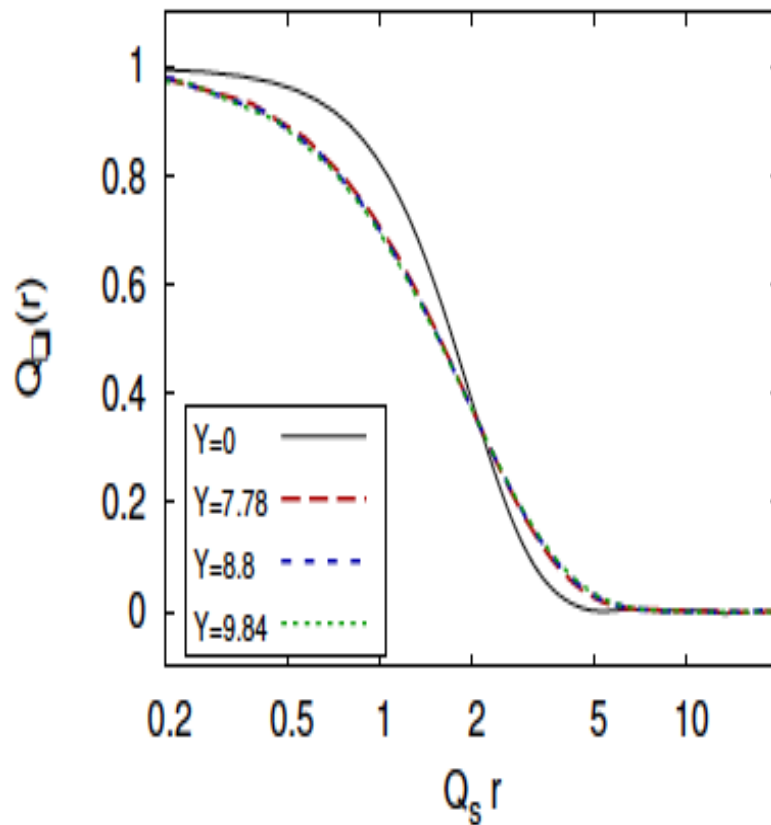


# Quadrupole evolution

comparing with Gaussian



# Quadrupole evolution



*Geometric scaling also present in quadrupoles*

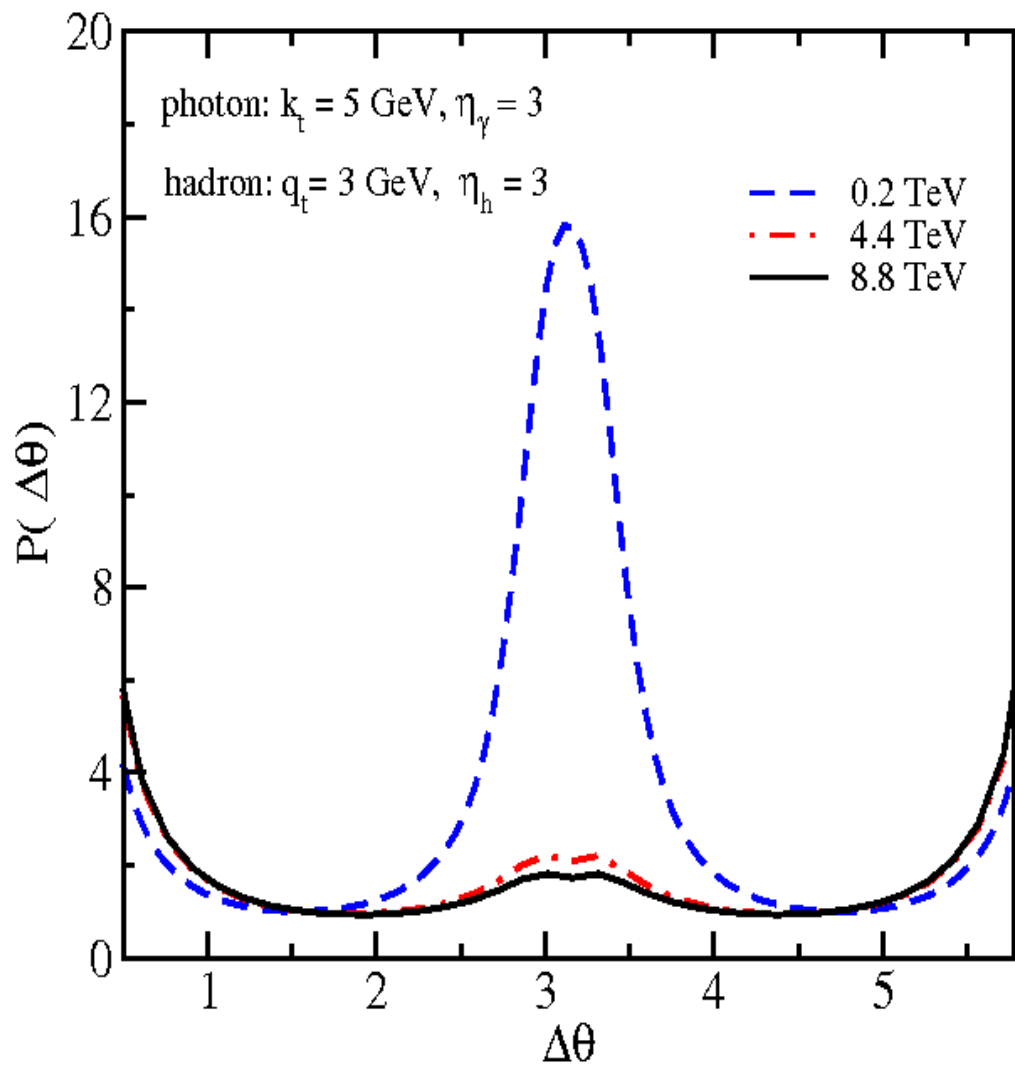
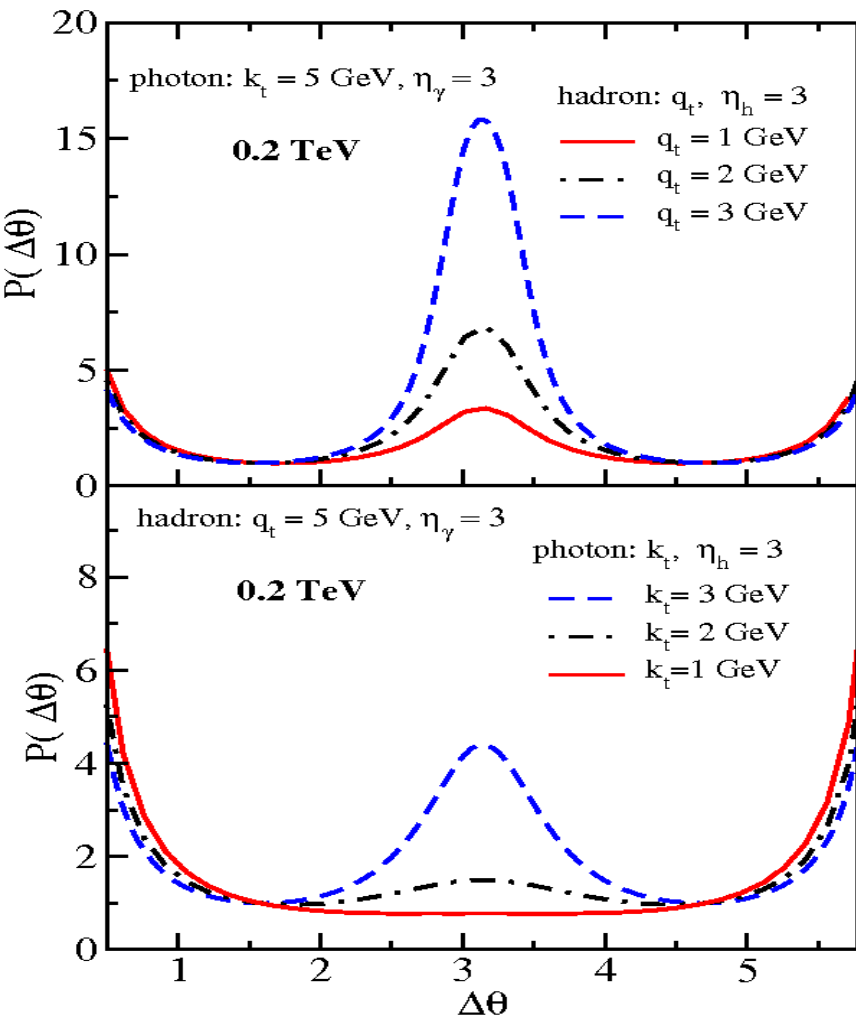
*Growth of the saturation scale*

***A simpler correlation to probe CGC:  
photon-hadron azimuthal correlation***

*based on arXiv:1204.1319  
J. Jalilian-Marian and A. Rezaeian*

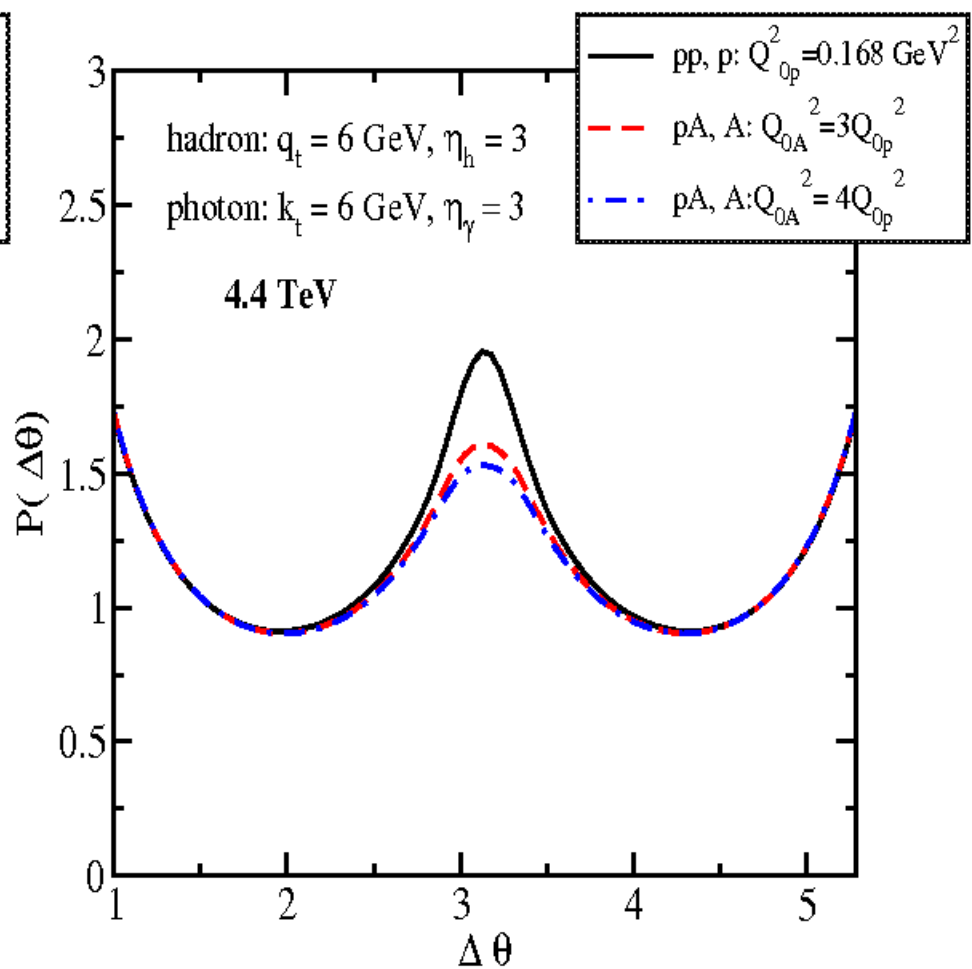
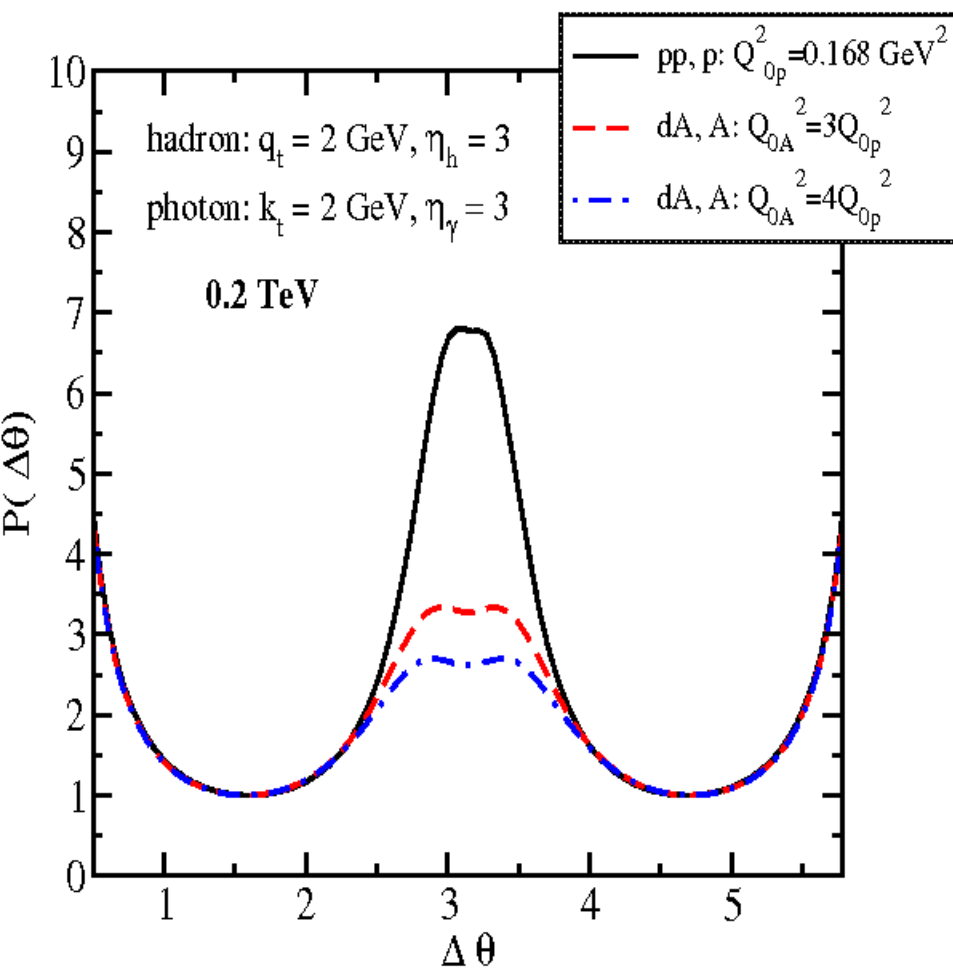
# photon-hadron azimuthal correlations

$$P(\Delta\theta) = \frac{d\sigma^{p(d) T \rightarrow h(q) \gamma(k) X}}{d^2\vec{b}_t dk_t^2 dq_t^2 dy_\gamma dy_l d\theta} [\Delta\theta] / \frac{d\sigma^{p(d) T \rightarrow h(q) \gamma(k) X}}{d^2\vec{b}_t dk_t^2 dq_t^2 dy_\gamma dy_l d\theta} [\theta = \theta_c]$$



# Centrality dependence

$$P(\Delta\theta) = \frac{d\sigma^{p(d) T \rightarrow h(q) \gamma(k) X}}{d^2\vec{b}_t dk_t^2 dq_t^2 dy_\gamma dy_l d\theta} [\Delta\theta] / \frac{d\sigma^{p(d) T \rightarrow h(q) \gamma(k) X}}{d^2\vec{b}_t dk_t^2 dq_t^2 dy_\gamma dy_l d\theta} [\theta = \theta_c]$$



# Photon production and photon-hadron correlations

*new processes to probe the dynamics of high energy QCD*

*suppression* of prompt photon spectrum  
in forward rapidity in  $p(d)A$

*disappearance of the away side peak* in  
photon-hadron azimuthal correlations  
in  $p(d)A$

*need to measure these at RHIC/LHC*

# The role of initial conditions

*McLerran-Venugopalan (93)*  $\langle \mathbf{O}(\rho) \rangle \equiv \int \mathbf{D}[\rho] \mathbf{O}(\rho) \mathbf{W}[\rho]$

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int d^2 \mathbf{x}_t \frac{\rho^a(\mathbf{x}_t) \rho^a(\mathbf{x}_t)}{2 \mu^2}} \quad \mu^2 \equiv \frac{g^2 A}{S_{\perp}}$$

$$\mathbf{T}(\mathbf{r}_t) \equiv \frac{1}{N_c} \langle \text{Tr} [1 - \mathbf{V}(\mathbf{r}_t)^\dagger \mathbf{V}(0)] \rangle \sim 1 - e^{-[\mathbf{r}_t^2 Q_s^2]^\gamma \log(e + \frac{1}{r_t \Lambda_{\text{QCD}}})}$$

*with*  $\gamma = 1.119$

*how about higher order terms in  $\rho$ ?*

$$\mathbf{W}[\rho] \simeq \mathbf{e}^{-\int d^2 \mathbf{x}_t \left[ \frac{\rho^a(\mathbf{x}_t) \rho^a(\mathbf{x}_t)}{2 \mu^2} - \frac{d^{abc} \rho^a(\mathbf{x}_t) \rho^b(\mathbf{x}_t) \rho^c(\mathbf{x}_t)}{\kappa_3} + \frac{F^{abcd} \rho^a(\mathbf{x}_t) \rho^b(\mathbf{x}_t) \rho^c(\mathbf{x}_t) \rho^d(\mathbf{x}_t)}{\kappa_4} \right]}$$

*these higher order terms make the single inclusive spectra steeper and give leading  $N_c$  correlations (ridge)*

*Dumitru-Jalilian-Marian-Petreska, PRD84 (2011) 014018*

*Dumitru-Petreska,, arXiv:1112.4760 [hep-ph]*

# Two-hadron angular correlations

*A unique window to dynamics of high energy QCD*

*We have just started to scratch the surface: there is much more to be understood*